UNIT 6 - KEPLER'S LAWS AND THE ORBITS OF MOONS

Introduction

Although first developed to describe the motion of the planets around the Sun, the third of Kepler's Laws of Planetary Motion can be used to relate the mass of a planet to the orbital radii and periods of its moons. If you know the orbital period of at least one moon, and the radius of its orbit, then you can obtain a value for the mass of the planet.

Kepler's Third Law

When a moon of mass m orbits a planet of mass M, then the force of gravity between them is given by Newton's Universal Law of Gravity:

$$F_g = \frac{GMm}{r^2}$$

This force is responsible for providing the centripetal acceleration required to keep the moon in its orbit. A direct consequence of Newton's Universal Law of Gravity is Kepler's Third Law:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

where r is the radius of the moon's orbit in meters, T is the orbital period in seconds (the time taken for the moon to orbit the planet once), M is the mass of the planet in kg, and G is the gravitational constant equal to 6.67×10^{-11} Nm^2/kg^2.

Note that for a given planet, the right side of the equation remains constant and Kepler's Third Law can be written as:

$$\frac{r^3}{T^2} = \text{constant}$$

The Galilean Moons of Jupiter

Select *Animation of the Galilean Moons of Jupiter* from the list of exercises. This simulation displays the orbital motion of the four Galilean moons of Jupiter - Io, Europa, Ganymede, and Callisto. The simulation shows the four Galilean moons from both a "face-on" perspective above the orbit of the moons, and an edge-on perspective as seen through a telescope on Earth. Note that from the edge-on perspective the pattern of moons is constantly changing, sometimes with two moons on either side of the planet, sometimes with three on one side and only one on the other, and sometimes with all the moons on one side. Occasionally not all four moons are visible, this occurs when one or more moons lie directly along the line of sight from Earth to Jupiter.

The date and time is displayed below the animation, allowing the orbital period to be determined for each moon. By selecting *Date* from the overhead menu, and then choosing *Enter a New Date and*

Time, any desired date and time from the past or future may be entered. You might find this feature helpful in identifying individual Galilean moons after having viewed them through a telescope.

The *Distances* window displays the current distance separating each moon from Jupiter as measured by an observer on Earth. From this edge-on perspective, the separation distance is observed to change as each moon appears to draw close to Jupiter and then move away as it completes an orbit Note that the true orbital radius for each moon is given by the separation distance that is observed when the moon has traveled to the extreme right or extreme left position.

JUPITER'S MOONS AND KEPLER'S LAWS

Gather orbital period and orbital radius data for all four Galilean moons and fill in the data table below. When determining the orbital period, you'll find it helpful to pause the simulation using options found on the *Animation* menu in order that the days/hours/minutes may be recorded at the beginning and end of a single orbit. Be careful to keep consistent time and distance units. (Note: For this portion of the exercise it may be more convenient to work with units of km and hours, rather than meters and seconds - it only matters that the units remain consistent.)

Name of Moon	**orbital radius (r)** (meters)	**orbital period (T)** (seconds)	**r^3/T^2**
Io			
Europa			
Ganymede			
Callisto			

How does the r^3/T^2 term compare among the four moons? Is this in agreement with Kepler's Third Law?

DETERMINATION OF JUPITER'S MASS

Using the orbital radius and orbital period data from one of the Galilean moons, calculate the mass of Jupiter using equation (2). For this calculation it is important that the orbital radius be expressed in meters and the orbital period expressed in seconds.

Mass of Jupiter = ________________kg