PHYS 2212

Look over
Chapter 31
sections 1-4, 6, 8, 9, 10, 11
Examples 1-8

PHYS 1112

Look over
Chapter 21
sections 11-14
Examples 16-18

Good Things To Know

1) How AC generators work.
2) How to find the two types of Reactance.
3) How to draw Phasor diagrams.
4) How to find the Impedance for a RLC circuit.
5) How to find the Current and Phase constant in a RLC circuit.
6) How to find the Power and the power factor in a RLC circuit.
7) The relations for a transformer.

Alternating Current Circuits

Just like how an isolating spring's energy will change from PE to KE, the energy in a circuit that has an inductor and a capacitor will be transferred between the magnetic field an the electric filed.

Electromagnetic Oscillations

A circuit with both an inductance (L) and a capacitance (C) is said to oscillate and the resulting oscillations of the capacitor's electric field and the inductors magnetic field are said to undergo Electromagnetic Oscillation

Energy Gets Pushed Around

Where the energy stored in the electric field is given by:

$$U_E = \frac{q^2}{2C}$$

and the energy stored in the magnetic field is:

$$U_B = \frac{Li^2}{2}$$

Where:

$$q = Q\cos(\omega t + \phi)$$

$$i = -\omega Q\sin(\omega t + \phi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

Damped Oscillations in an RLC Circuit

A circuit containing resistance, inductance, and capacitance is called an **RLC Circuit**.

With a resistance R present, the total electromagnetic energy U of the circuit is no longer constant; instead it decreases with time as the energy is transformed to thermal energy in the resistance.

$$U_E = \frac{q^2}{2C} = \frac{Q e^{-(Rt/2L)} \cos(\omega' t + \phi)}{2C}$$

Alternating Current

In most countries the energy is supplied by an oscillating emfs and currents or Alternating Current (AC).

The basic advantage of AC is that as the current alternates, so does the magnetic field that surrounds the conductor. This makes the operation of rotating machinery such as generators and motors easier.

AC Generator

A simple model of an AC generator is a conducting loop forced to rotate through an external magnetic field B.

The induced emf $\mathcal{E}$ will very according to the to the angular speed that the loop is rotating at as:

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$

and an induced current given as

$$I = I \sin(\omega_d t - \phi)$$

Resistors in AC Circuits

Looking at a circuit with only an AC power source and a resistor then we can use the loop rule to write:

$$\mathcal{E} - v_R = 0$$

so

$$v_R = \mathcal{E}_{max} \sin(\omega_d t)$$
$$v_R = V_R \sin(\omega_d t)$$

But Ohm Said

Now using Ohm's Law we can get the current as:

$$i_R = \frac{V_R}{R} \sin(\omega_d t)$$
$$i_R = I_R \sin(\omega_d t)$$

Capacitors in AC Circuits

Like we did with the resistors we can write down the potential difference across the capacitor as:

$$v_C = V_C \sin(\omega_d t)$$

Current and Capacitance

Where from the definition of capacitance we can write:

$$q_C = Cv_R$$
$$q_C = CV_R \sin(\omega_d t)$$

We can now find the current using:

$$i_c = \frac{dq_c}{dt}$$

Capacitive Reactance

$$i_C = \omega_d C V_C \cos(\omega_d t)$$
$$i_C = \frac{V_C}{X_C} \cos(\omega_d t)$$

We can now define the capacitive reactance:

$$X_c = \frac{1}{\omega_d C}$$

So now the relation between the current and voltage takes on a form like Ohm's law:

$$V_C = I_C X_C$$

Inductors in AC Circuits

As we have seen before that the voltage across an inductor can be written as

$$v_L = V_L \sin(\omega_d t)$$

Inductors and Current

From Faradays Law we can write:

$$V_L = L\left(\lim_{\Delta t \to 0} \frac{\Delta i}{\Delta t}\right)$$

$$V_L = L\frac{di}{dt}$$

Inductive Reactance

$$i_L = -\frac{V_L}{\omega_d L}\cos(\omega_d t)$$

$$i_L = -\frac{V_L}{X_L}\cos(\omega_d t)$$

We can now define the Inductive reactance:

$$X_L = \omega_d L$$

So now the relation between the current and voltage takes on a form like Ohm's law:

$$V_L = I_L X_L$$

Putting It all together

We now are ready to apply the alternating emf:

$$\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$

to the full RLC circuit because R, L, C, are in series, the same current will pass through each of them:

$$i = I\sin(\omega_d t - \phi)$$

where ϕ is a phase constant that we will need to find a value for.

Set Phasors on Rotation

To make the solution clearer we will use phasor Diagrams.

The first phasor diagram shows the current at a time t.

Phasor Diagrams

The next phasor diagram represents the voltages across R, L, and C at the time t. The phasors in the diagram are measured with respect to I using the following:

Phasors

Resistor–Here the current and the voltage are in phase: so the angle of rotation for the voltage phasor is the same as that of the phasor I.

Capacitor–Here the current leads the voltage by 90°; so the angle of rotation of the voltage phasor V_C is 90° less then that of the phasor I.

Inductor–Here the current lags the voltage by 90°; so the angle of rotation of the voltage phasor V_L is 90° greater then that of the phasor I.

Adding up the Voltages

The last phasor diagram shows the phasor representing the applied emf.

Thus at any time t the projection E is equal to the algebraic sum of the projections v_R v_C and v_L.

$$E = v_R + v_C + v_L$$

Current

This means that the phasor E_{max} must be equal to the vector sum of the three voltage phasors V_R V_C and V_L

$$E_{max}^2 = V_R^2 + (V_L - V_C)^2$$

or if we use R, X_L and X_C we get:

$$E_{max}^2 = IR^2 + (IX_L - IX_C)^2$$

Then we can get the current as:

$$I = \frac{E_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance

We can now define the **Impedance Z** for the circuit as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

So we can now write down a version of Ohm's Law:

$$I = \frac{E_{max}}{Z}$$

The Phase Constant

We have now reached one of our goals the: the current in terms of the circuit elements.

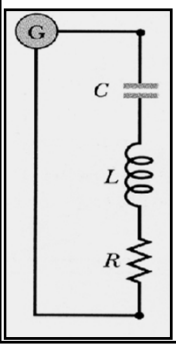
$$I = \frac{E_{max}}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

The phase constant ϕ for the circuit can now be define as:

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{X_L - X_C}{X_R}$$

Example 1



1) A series RLC ac circuit has *R=425 Ω, C=3.50 μF, L=1.25 H, ω=377 rad/s* and E_{max}*=150.0 V.*

a) What is the total impedance?
b) What is the maximum current?
c) What is the phase angle?
d) What is the maximum Voltage and the instantaneous voltage across each element.

Power in AC Circuits

In the RLC circuit the source of energy is the alternating-current generator.

Some of the energy that it provides is stored in the magnetic field in the inductor, some is stored in the electric field of the capacitor, and some is dissipated as thermal energy in the resistor. In steady-state operation the average energy stored in the capacitor and in the inductor remains constant.

Rate of Power Usage

The net transfer of energy is thus from the generator to the resistor.

The rate at which energy is dissipated in the resistor is:

$$P = i^2 R = \left[I \sin(\omega_d t - \phi) \right]^2 R$$
$$= I^2 \sin^2(\omega_d t - \phi) R$$

RMS Current

$$P_{ave} = \frac{I^2 R}{2} \text{ or } \left(\frac{I}{\sqrt{2}} \right)^2 R$$

Where we can call the current term:

$$I_{rms} = \frac{I}{\sqrt{2}} \text{ so } P_{ave} = I_{rms}^2 R$$

Power Factor

we can also define the rms value for the voltage and emf

$$V_{rms} = \frac{V}{\sqrt{2}} \qquad \mathcal{E}_{rms} = \frac{\mathcal{E}}{\sqrt{2}}$$

We can also write:

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$$

so we can rewrite:

$$P_{ave} = \frac{\mathcal{E}_{rms}}{Z} I_{rms} R = \mathcal{E}_{rms} I_{rms} \frac{R}{Z}$$
$$\text{but } \frac{R}{Z} = \cos\phi$$

$$P_{ave} = \mathcal{E}_{rms} I_{rms} \cos\phi$$

where $\cos\phi$ is called the power factor.

Example 2

2) Calculate the average power delivered to the series RLC circuit from Example 1.

Example 3

3)Consider a *735 kV* line used to transmit electric energy from the La Grande 2 hydroelectric plant in Quebec to Montreal, *1000 km* away. If the current is *500 A* and the power factor is close to unity. What percent is the average rate that energy is dissipated to the resistance in the wire if the wire has a resistance of *0.220 Ω/m*?

Transformers

As we saw in the previous example the general energy transmission rule:
Transmit at the highest possible voltage and the lowest possible current.

So we need a device with which we can raise (for transmission) and lower for use the voltage in a circuit.

The transformer is such a device.

Ideal Transformers

The ideal transformer consists of two coils, with different numbers of turns, wound around an Iron core.

The primary winding, of N_p turns, is connected to an alternating-current generator with an alternating emf.

The secondary winding, of Ns turns, is connected to a load resistance R.

Primary and Secondary Relationships

From Faradays law of induction the induced emf per turn is the same for the primary and secondary so:

$$\frac{V_p}{N_p} = \frac{V_s}{N_s} \text{ so } V_s = V_p \frac{N_s}{N_p}$$

using $I_p V_p = I_s V_s$ and conservation of energy we get:

$$I_s = I_p \frac{N_p}{N_s}$$

Example 4

4) A step–down transformer is used for recharging the batteries of portable devices such as tape players. The turns ratio inside the transformer is *13:1*, and it is used with *120 V* (rms) household service. If a particular ideal transformer draws *0.350 A* from a house outlet, what

a) voltage and
b) current are supplied to a tape player from the transformer?
c) How much power is delivered?