MATH 1111 PRACTICE TEST 1 FALL 2009 ANSWERS

0. (2 points if it is printed neatly)
Name:____
Note that anything typed in blue is commentary and you are not expected to include the commentary when you take the test. In order to show your work you write something similar to what is given in black!

1. (4 points) Determine if the following equations define *y* as a function of *x*. You must justify your answers.

(a) $y^2 = 4x + 1$

Let $x = 2$

$y^2 = 4(2) + 1$

Then $y^2 = 9$

$y = \pm 3$

Since there are two *y*-values for $x = 2$ the equation does not define a function

(b) $y = x^2 - x - 1$

There will only be one *y*-value for each *x* value, so the equation does define a function.

2. (4 points) Given $g(x) = x^2 - 3x$ evaluate and simplify each of the following:

(a) $g(2)$

$$g(2) = (2)^2 - 3(2) = 4 - 6 = -2$$

(b) $g(-3)$

$$g(-3) = (-3)^2 - 3(-3) = 9 + 9 = 18$$

(c) $g(x - c)$

$$g(x-c) = (x-c)^2 - 3(x-c) = x^2 - 2xc + c^2 - 3x + 3c$$

3. (3 points) Find values of *x* such that $f(x) = 0$ if $f(x) = 3x + 15$

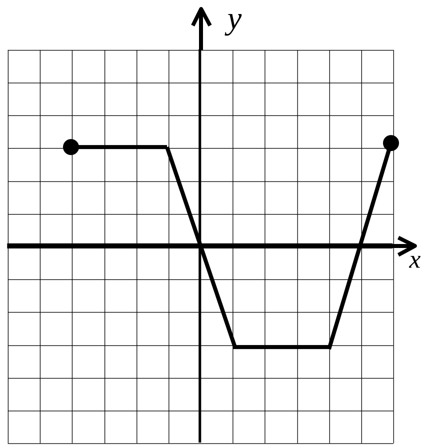
$$3x + 15 = 0$$

$$3x = -15$$

$$x = -5$$

4. (6 points) The following is the graph of a function *f*.

(a) State the domain and range and find the indicated function values.



Domain: [–4, 6]

Range: [–3, 3]

$f(-2) =$

You observe that (–2, 3) is a point on the graph so, $f(-2) = 3$

$f(1) =$

You observe that (1, –3) is a point on the graph so, $f(1) = -3$

$f(5)$

You observe that (5, 0) is a point on the graph so, $f(5) = 0$

(b) Find the values of *x* such that $f(x) = 0$

You look for the values of *x* where the graph crosses the *x*-axis.

$x = 0, \ 5$

5. (6 points) Determine if the following functions are odd, even or neither.

You figure $f(-x)$ and $-f(x)$. If $f(-x) = f(x)$ then *f* is even. If $f(-x) = -f(x)$ then *f* is odd. If $f(x), f(-x)$ and $-f(x)$ are all different then *f* is neither even nor odd.

(a) $f(x) = x^3 + x + 7$

$f(-x) = (-x)^3 + (-x) + 7 = -x^3 - x + 7$

$-f(x) = -(x^3 + x + 7) = -x^3 - x - 7$

f is neither

(b) $g(x) = -x^3 + x$

$g(-x) = -(-x)^3 + (-x) = -(-x^3) - x = x^3 - x$

$-g(x) = -(-x^3 + x) = x^3 - x$

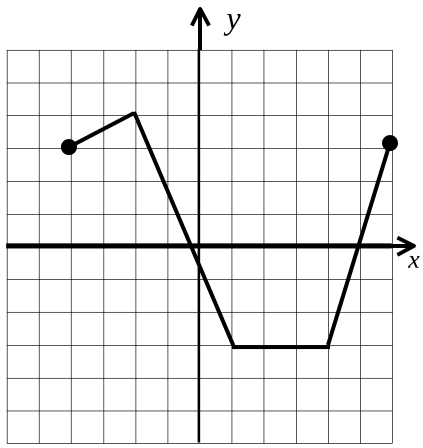
g is odd

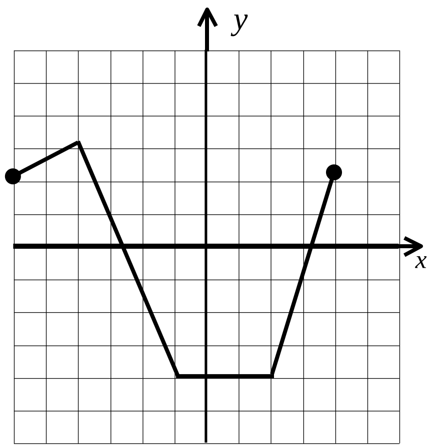
(c) $h(x) = x^4 + 4x^2 - 8$

$h(-x) = (-x)^4 + 4(-x)^2 - 8 = x^4 + 4x^2 - 8$

h is even (There is no need to look at $-h(x)$ when *h* is even)

6. (5 points) The following is the graph of a function *f*. State the operations you perform.





Sketch the graph of the function $G(x) = f(x+2) - 1$

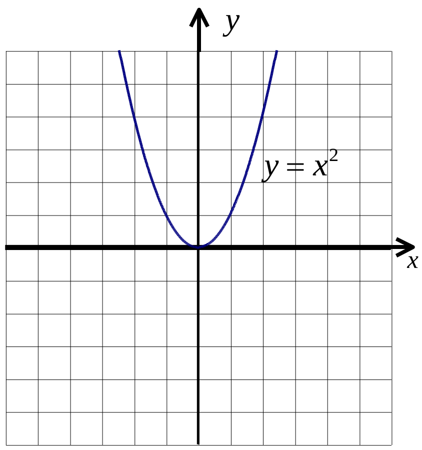
1. Replace *x* by *x* + 2; this shits the graph 2 units left.

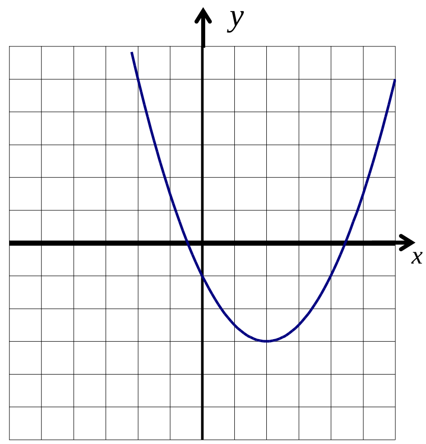
2. Subtract 1. This shifts the graph down 1 unit

7. (5 points) Starting with the graph of $y = x^2$ graph $g(x) = \frac{1}{2}(x-2)^2 - 3$ using the techniques of shifting and stretching. **State clearly which operations you perform.**

Replace *x* by *x* – 2; $f(x-2) = (x-2)^2$ This shifts the graph two units right.
Multiply by $\frac{1}{2}$; $\frac{1}{2} f(x-2) = \frac{1}{2}(x-2)^2$ This compresses the graph to half its height

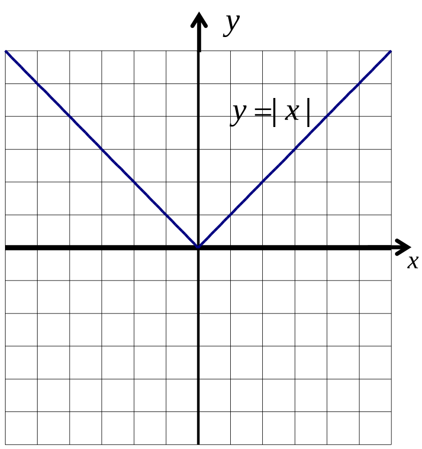
Subtract 3; $\frac{1}{2} f(x-2) - 3 = \frac{1}{2}(x-2)^2 - 3$ This shifts the graph down three units

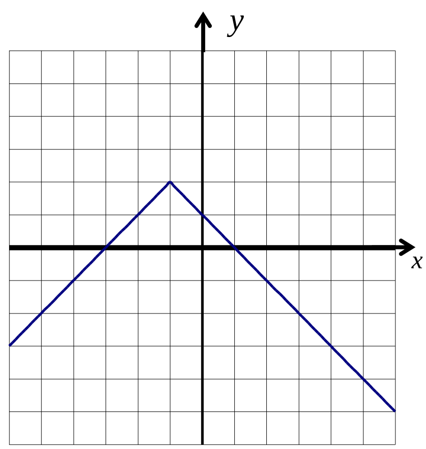




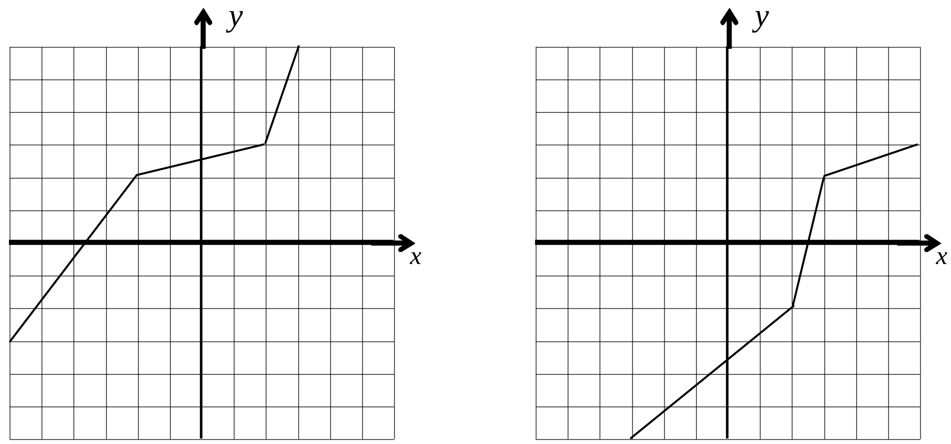
8. (5 points) Starting with the graph of $y = |x|$ graph $g(x) = -|x+1| + 2$ using the techniques of shifting and stretching. **State clearly which operations you perform.**

Replace x by x + 1; $f(x+1) = |x+1|$. This shifts the graph one unit left
Multiply by −1; $-f(x+1) = -|x+1|$. This reflects the graph in the x-axis
Add 2; $-f(x+1) + 2 = -|x+1| + 2$. This shifts the graph up two units





9. (5 points) The following is the graph of a function $f(x)$. Sketch the graph of $f^{-1}(x)$



If (a, b) is on the graph of $f(x)$ then (b, a) is on the graph of $f^{-1}(x)$. The graph of $f(x)$ consists of straight lines joining (–6, –3), (–2, 2), (2, 3) and (3, 6). So, the graph of $f^{-1}(x)$ consists of straight lines joining (–3, –6), (2, –2), (3, 2) and (6, 3).

10. (5 points) Find the inverse of $f(x) = \dfrac{2x+3}{x+2}$

Let $y = \dfrac{2x+3}{x+2}$

Interchange x and y and then solve for y.

$$x = \frac{2y+3}{y+2}$$

$$x(y+2) = 2y+3$$

$$xy + 2x = 2y+3$$

$$xy - 2y = 3 - 2x$$

$$y(x-2) = 3-2x$$

$$y = \frac{3-2x}{x-2}$$

You have now found a formula for $f^{-1}(x)$

$$f^{-1}(x) = \frac{3-2x}{x-2}$$