New Graphs from Old

Suppose you know what the graph of $y = f(x)$ looks like. Suppose $c > 0$. The idea is to develop some techniques for graphing functions that can somehow be expressed using f.

1. The graph of $y = f(x - c)$ is the graph of $f(x)$ shifted c units right.

2. The graph of $y = f(x) + c$ is the graph of $f(x)$ shifted c units up.

3. The graph of $y = f(x + c)$ is the graph of $f(x)$ shifted c units left.

4. The graph of $y = f(x) - c$ is the graph of $f(x)$ shifted c units down.

To help you remember how it works note that a change outside of $f(x)$, as in 2 or 4, results in a change in the vertical direction. A change inside the definition of $f(x)$, as in 1 or 3, results in a change in the horizontal direction.

Note also that changes made inside $f(x)$ have the "opposite" effect of changes made outside of $f(x)$. For example, adding c outside shifts the graph upwards, or in the positive direction; whereas adding c on the inside shifts the graph left, or in the negative direction. Vertical changes are usually easier to see intuitively, so if you think vertical first it will be easier to see what happens horizontally!

5. The graph of $y = cf(x)$ is the graph of $f(x)$ stretched or compressed vertically. It is compressed if $c < 1$ and it is stretched if $c > 1$.

6. The graph of $y = f(cx)$ is the graph of $f(x)$ stretched or compressed horizontally. It is stretched if $c < 1$ and it is compressed if $c > 1$.

7. The graph of $y = -f(x)$ is the graph of $f(x)$ reflected across the x-axis.

8. The graph of $y = f(-x)$ is the graph of $f(x)$ reflected across the y-axis.

Problems often involve more than one transformation. When they do you need to figure how to get from the initial graph to the graph you want using the above steps one at a time

Example: You know what the graph of $y = |x|$ looks like. How do you get the graph of $y = 2|x + 3| - 4$.

Start with $y = |x|$

Step 1: $y = |x + 3|$ Shift 3 units left

Step 2: $y = 2|x + 3|$ Stretch vertically by a factor 2.

Step 3: $y = 2|x + 3| - 4$ Shift down 4 units