Lesson 2.2.1: What Is Average?
Theme: Personal Finance

Making Connections to the Lesson
(1) Which of the following was one of the main mathematical ideas of the lesson?
   (i) Any of the three measures of central tendency (mean, median, and mode) are good representations of data. It does not matter which one you use.
   (ii) U.S. college students carry far too much credit card debt.
   (iii) The mean is calculated by adding all the numbers and dividing by the number of data points.
   (iv) The mean, median, and mode all give important information about a data set, but they do not give a complete picture of the data set.

(2) In Lesson 2.1.4, you learned about reading graphs. Describe a connection between interpreting a graph and interpreting measures of central tendency. You can use one of the sentence stems given below if you wish.

   Question number______ in Lesson 2.1.4 connects to Question number ______ in Lesson 2.2.1 because...

   The idea of ______________________ in Lesson 2.1.4 connects to Lesson 2.2.1. An example of the connections is...

Developing Skills and Understanding
(3) Use the following data set to answer the questions.

   13  15  20  20  20  20  20  23  27  31

   (a) What is the mean?
   (b) What is the mode?
   (c) What is the median?
   (d) What fraction of the numbers in the data set are less than the median?
   (e) What fraction of the numbers in the data set are greater than the median?
   (f) Which of the following statements are correct?
      (i) The median is the middle of a data set. Half of the data points are always less than the median, and half are always greater than the median.
      (ii) The median is the middle of a data set. Half of the data points are either less than or equal to the median.
      (iii) The median is the middle of a data set. At least half of the data points are always equal to the median.
      (iv) The median is not the middle of a data set. You cannot predict the distribution of the numbers in relationship to the median.
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(4) Consider the statement “Worldwide, there are more than $2.5$ trillion in credit card transactions annually.”

(a) What is the daily average dollar amount of transactions? Round to the nearest hundred million dollars.

(b) How many dollars in credit card transactions are made each day?

   (i) $6,849,315$

   (ii) $6,849,315,068$

   (iii) $1,460,000,000$

   (iv) $1,460,000$

   (v) It is impossible to know.

(5) Students at Dover Community College (DCC) have a mean credit card debt of $3,600 with a median of $1,500. Students at Ralton Community College (RCC) have a mean credit card debt of $3,000 with a median of $2,800. Which statements about the two groups are true based on this information? There may be more than one correct answer.

   (i) Less than half of DCC students have debt in excess of $3,600.

   (ii) No more than half of RCC students have debt less than $2,800.

   (iii) About three-fourths of DCC students have debt less than $3,600.

   (iv) The largest debt of the RCC students is less than the largest debt of the DCC students.

   (v) The total debt of RCC students is less than the total debt of DCC students.

   (vi) The total debt of RCC students is larger than the total debt of DCC students.

(6) Decide whether the following statements must be true or not, based on the information provided. If the statement must be true, write True; otherwise, write False. Be prepared to explain your reasoning.

   (a) The median of 25 numbers is 13. Twelve of the numbers must be greater than 13.

   (b) The average of 11 numbers is 130. None of the 11 numbers are more than 260.

   (c) The average of 25 numbers is 100, and the median of those 25 numbers is also 100. The mode of the 25 numbers must be 100.

   (d) The mean of 45 numbers is 70. If you pick any group of 10 numbers from the 45, the mean will be 70.

   (e) The average of 42 numbers is 20. The sum of all 42 numbers is 840.

   (f) The average of 49 numbers is 100. If a 50th number is added and the average remains at 100, the 50th number must have been 100.
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(7) Rio Blanca City Hall publishes the following statistics on household incomes of the town’s citizens. The mode is given as a range.

Mean: $257,000  Median: $65,000  Mode: $20,000–$30,000

Which measure would be the most useful for each of the following situations?

(a) State officials want to estimate the total amount of state income tax paid by the citizens of Rio Blanca.

(b) The school district wants to know the income level of the largest number of students.

(c) A businesswoman is thinking about opening an expensive restaurant in the town. She wants to know how many people in town could afford to eat at her restaurant.

Making Connections Across the Course

(8) Use Figures 1 and 2 for the following questions.

(a) Shade 40% of each figure.
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(b) Which of the following statements are correct? There may be more than one correct answer.

(i) The shaded area of Figure 1 is larger than the shaded area of Figure 2 because Figure 1 is larger than Figure 2.

(ii) The shaded area of Figure 1 is the same as the shaded area of Figure 2 because they are both 40% of the square.

(iii) The shaded area of Figure 1 is the same proportion of the figure as the shaded area of Figure 2 because they are both 40% of the square.

(c) These figures illustrate what important concept?

(i) Percentages cannot be used for comparisons unless the reference values are equal.

(ii) Percentages compare measures relative to the size of the reference values, but do not give information about absolute measures.

(iii) Percentages are a ratio out of 100, so they can always be compared directly. In other words, 60% of one value is equal to 60% of another value.

(9) Animal population densities are important to those who manage wildlife. This is particularly true when animals pose risks to humans. Managing the population of grizzly bears in Yellowstone National Park is one example. Grizzly bears roam Yellowstone Park, which is 3,472 square miles. Additionally, the bears roam a surrounding area that is 40% as large as Yellowstone.

Each male grizzly bear needs a territory to roam that is about 300 square miles, while each female bear needs a roaming territory of about 100 square miles. Although their territories overlap considerably, each bear needs at least 10 square miles to himself/herself to call home.¹

For the following questions, assume that bears roam freely within and outside Yellowstone without being captured or shot.

(a) Calculate the total available land in and around Yellowstone for grizzly bears to roam. Round to the nearest square mile.

(b) Estimate the maximum number of bears that can be supported on the available land.

(c) If there are about 1.5 adult female bears for every adult male bear, then approximately how many male bears can live in and around Yellowstone Park? How many female bears?

The bar graph below shows the number of bear sightings (all bears, not only grizzlies) in Yellowstone National Park from 1998 to 2002.² Because the number of sightings is affected by the number of visitors to the park, these data are shown as well.³

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Note: Because the number of visitors each year is much larger than the number of bear sightings, the data are plotted on different scales. The scale on the left (ranging from 0 to 3,500,000) is for the number of visitors, shown in gray. The scale on the right (ranging from 0 to 980) is for the number of bear sightings, shown in black.

(d) In what year was the percentage of visitors who saw bears the highest?

(e) In what two years was the percentage of visitors who saw bears the lowest?

(f) Estimate the average number of bear sightings over these five years.

(i) 600
(ii) 700
(iii) 800
(iv) 900
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Preparing for the Next Lesson (2.2.2)
(10) Answer the questions for the graph shown below.

(a) What was the average or mean price of a new home in 2005?
(b) What was the median price of a new home in 1984?
(c) Which of the following intervals had the largest increase in the median price?
   (i) 1965–1968
   (ii) 1981–1984
   (iii) 1989–1992
   (iv) 1997–2000
(d) In what two-year period was the largest drop in the average price?
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(11) You should be able to do the following things for the next class. Rate how confident you are on a scale of 1–5 (1 = not confident and 5 = very confident).

Before beginning Lesson 2.2.2, you should understand the concepts and demonstrate the skills listed below:

<table>
<thead>
<tr>
<th>Skill or Concept: I can ...</th>
<th>Rating from 1 to 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform basic operations using quantities as integers, fractions, or decimals with the aid of technology.</td>
<td></td>
</tr>
<tr>
<td>Find the mean, median, and mode of a set of numeric data.</td>
<td></td>
</tr>
<tr>
<td>Read a line graph.</td>
<td></td>
</tr>
</tbody>
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